THE MIXING OF TWO GAS FLOWS MOVING IN COAXIAL PIPES SEPARATED BY A PERFORATED WALL

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ABSTRACT: The mixing of two gases possessing different physical and thermodynamic properties in coaxial cylindrical pipes separated by a perforated wall is considered in a one-dimensional scheme. Mass transfer caused by the difference in static pressures in the flows takes place between the media. It is assumed that the gases in the pipe are mixed instantaneously and that thermodynamic equilibrium is established in the mixture of gases. It is also assumed that friction and heat transfer on the walls of the pipes are negligibly small as compared with the mixing effect. When setting up the momentum equation for the motion in the pipe where mixing occurs, the mutual direction of the mixing flows and the angle of incidence of the stream are taken into consideration.

The one-dimensional equations of steady motion of two gas flows separated by a perforated wall are of the form

$$dQ_2 = -dQ_1, \quad d(Q_2 i_2) = -i_1 dQ_1, \quad Q_j = f_j u_j \rho_j, \quad (1)$$
$$d(Q_2 u_2) + f_2 dp_2 = \beta\,[d(Q_1 u_1) + f_1 dp_1], \quad i_j = c_{pj} T_{j0}.$$

Here Q_j is the discharge through a cross section of the pipe, i_j the enthalpy, p_j the static pressure, u_j the velocity, ρ_j the density, T_{j0} the stagnation temperature, c_{pj} the specific heat at constant pressure; $j = 1, 2$, the subscript 1 referring to the pipe from which the gas is flowing and the subscript 2 referring to the pipe in which the mixture of gases flows. The cross-sectional area of the pipe f_j is constant along the axis.

In the momentum equation, the parameter β takes account of the mutual directionality of the flows in pipes 1 and 2 and the part of the momentum of the gas flowing from pipe 1 transferred to the mixture of gases in pipe 2. Depending on the arrangement of the perforations, the angle of incidence of the gas to be mixed in pipe 2 varies, consequently, the part of the momentum transferred to the mixture of gases in the direction of the axis of the pipe also varies.

When arranging the perforations so as to ensure transfer of the momentum of the gas flowing through without losses (the direction of incidence of the flow in the perforated opening in pipe 1 and the direction of the stream into pipe 2 coincided with the direction of the axis of the pipes, and the losses in the apertures can be neglected), the parameter $\beta = -1$ in the case of flows in pipes 1 and 2 in the same direction and the parameter $\beta = +1$ in the case of oppositely directed flows. When the stream enters pipe 2 perpendicular to the direction of the flow, $\beta = 0$. Depending on the type of perforations and the mutual direction of the flows in pipes 1 and 2, the value of β ranges within limits $-1 \le \beta \le 1$.

The flow of gas from the first pipe into the second pipe obeyed the laws

$$\frac{dQ_1}{dx} = -\mu F(x)\left(\frac{2}{\varkappa_1+1}\right)^{\vartheta_1}\left(\frac{2\varkappa_1}{\varkappa_1+1}p_1\rho_1\right)^{1/2} \qquad \left(\vartheta_1 = \frac{1}{\varkappa_1 - 1}\right), \quad (2)$$

with a precritical drop in the static pressures

$$\frac{dQ_1}{dx} = -\mu F(x)\left\{\frac{2\varkappa_1}{\varkappa_1-1}p_1\rho_1\left[\left(\frac{p_2}{p_1}\right)^{2\theta} - \left(\frac{p_2}{p_1}\right)^{1+\theta}\right]\right\}^{1/2}$$
$$\left(\theta = \frac{1}{\varkappa_1}\right) \quad (3)$$

Here $F(x)$ is the area of the perforations in the partition per unit length of the pipe, and μ is the drag coefficient of the aperture.

The gases and the mixture of gases satisfy the Clapeyron equation

$$p_j = R_j \rho_j T_j. \quad (4)$$

For the mixture of gases, the value of the gas constant $R_2 = c_{p2} - c_{i2}$ and $\varkappa_2 = c_{p2}/c_{v2}$ is measured along the pipe and is determined through the parameters of the second gas before mixing $Q_2(0)$, $R_2(0)$, $\varkappa_2(0)$ and the value of the parameters of the gas to be mixed

$$R_2 = \frac{R_2(0) + nR_1}{1+n}, \qquad \varkappa_2 = \frac{\varkappa_2(0) + n\varkappa_1}{1+n}$$
$$\left(\frac{Q_1(0) - Q_1}{Q_2(0)} = n\right). \quad (5)$$

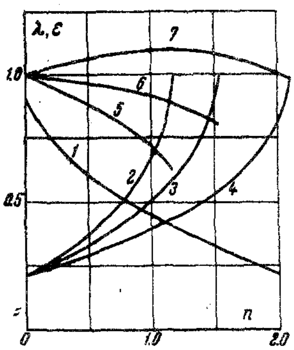

Fig. 1

The equations of motion (1) are integrated:

$$Q_1 + Q_2 = Q_1(0) + Q_2(0),$$
$$Q_1 i_1 + Q_2 i_2 = Q_1(0)\, i_1(0) + Q_2(0)\, i_2(0),$$
$$p_2 f_2 + Q_2 U_2 + \beta\,(p_1 f_1 + Q_1 u_1) = p_2(0)\, f_2 +$$
$$+ Q_2(0)\, u_2(0) + \beta\,[p_1(0)\, f_1 + Q_1(0)\, u_1(0)]. \quad (6)$$

The integrals of motion (6) and the equation of state (4) can be written in the form

$$q(\lambda_1) = q[\lambda_1(0)] - \frac{n\sqrt{\tau}}{\eta\sigma}\, q[\lambda_2(0)]\, B_1,$$
$$\psi = \left(\frac{\omega}{1+n}\right)^2, \qquad \varepsilon = \frac{q[\lambda_2(0)]}{q(\lambda_2)}\,\omega B_2,$$
$$z(\lambda_2) = \frac{1}{\omega B_2}\Big\{z[\lambda_2(0)] +$$
$$+ \beta\eta\sigma\left[z(\lambda_1)\frac{q(\lambda_1)}{q[\lambda_1(0)]} - z[\lambda_1(0)]\frac{q[\lambda_1(0)]}{q[\lambda_2(0)]}\right]\Big\}. \quad (7)$$

Here we have introduced the dimensionless quantities [1]:

$$\lambda_j = \frac{u_j}{a_j^*}, \qquad \tau = \frac{T_{10}(0)}{T_{20}(0)}, \qquad \sigma = \frac{p_{10}(0)}{p_{20}(0)}$$
$$\left(a_j^* = \left(\frac{2\varkappa_j}{\varkappa_j+1} R_j T_{j0}\right)\right)^{1/2},$$
$$\varepsilon = \frac{p_{20}}{p_{20}(0)}, \qquad \psi = \frac{T_{20}}{T_{20}(0)}, \qquad \eta = \frac{f_1}{f_2}.$$

The subscript 0 after the symbol $j = 1, 2$ denotes the total pressure and the stagnation temperature, respectively; the index (0) denotes the value of the parameters at the point $x = 0$ at the beginning of mixing. Moreover, the following notation is employed:

$$z(\lambda_j) = \lambda_j + \frac{1}{\lambda_j}, \qquad T(\lambda_j) = 1 - \frac{\varkappa_j - 1}{\varkappa_j + 1}\lambda_j^2,$$
$$q(\lambda_j) = \lambda_j\left[\frac{\varkappa_j+1}{2}\, T(\lambda_j)\right]^{\theta_j} \qquad \left(\theta_j = \frac{1}{\chi_j - 1}\right),$$

$$\omega = (1+n)\left[1 + n\tau\frac{c_{p1}}{c_{p2}(0)}\right]^{1/2}\left[1 + n\frac{c_{p1}}{c_{p2}(0)}\right]^{-1/2},$$

$$B_j = \left(\frac{2}{\varkappa_2(0)+1}\right)^{\chi_2}\left(\frac{2}{\varkappa_j+1}\right)^{\chi_j}\left(\frac{\varkappa_2(0)R_j}{\varkappa_j R_2(0)}\right)^{1/2}$$

$$\left(\chi_2 = \frac{\varkappa_2(0)+1}{2[\varkappa_2(0)-1]}, \quad \chi_j^* = -\frac{\varkappa_j+1}{2(\varkappa_j-1)}\right).$$

The integrals of motion (7) determine the value of the parameters λ_1, λ_2, ε, ψ depending on the change in the value of n for given values of $\lambda_1(0)$, $\lambda_2(0)$, σ, τ, $\varkappa_1$, $\varkappa_2(0)$, R_1, $R_2(0)$, β, and η.

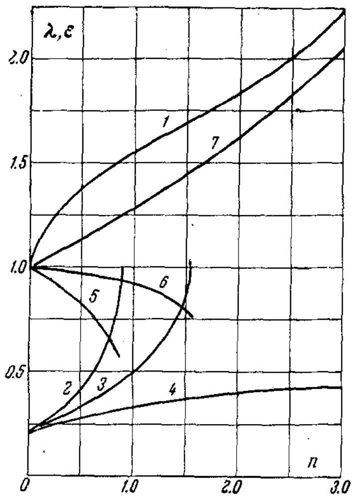

Fig. 2

Changes in n along the pipe depending on the distribution of perforations are determined by formulas (2), (3), which can be represented in the form

$$\int_0^n \frac{dn}{\Phi(n)} = \int_0^x \mu F(x)\, dx. \qquad (8)$$

For a supercritical drop in static pressures

$$\Phi(n) = -\left(\frac{2}{\varkappa_1+1}\right)^{\theta_1}\left(\frac{2\varkappa_1}{\varkappa_1+1}p_1\rho_1\right)^{1/2}\frac{1}{Q_2(0)} \qquad \left(\theta_1 = \frac{1}{\varkappa_1-1}\right).$$

For a precritical drop in static pressures

$$\Phi(n) = -\left\{\frac{2\varkappa_1}{\varkappa_1-1}p_1\rho_1\left[\left(\frac{p_2}{p_1}\right)^{2\theta_2} - \left(\frac{p_2}{p_1}\right)^{1+\theta_2}\right]\right\}^{1/2} \qquad \left(\theta_2 = \frac{1}{\varkappa_1}\right).$$

The quantities p_1, ρ_1, p_2 are determined as follows:

$$p_1 = p_{10}T^{1+\vartheta_1}(\lambda_1), \quad \rho_1 = \frac{p_{10}}{R_1 T_{10}}T^{\vartheta_1}(\lambda_1),$$

$$p_2 = \frac{p_{10}}{\varepsilon}T^{1+\vartheta_2}(\lambda_2)$$

$$\left(\vartheta_1 = \frac{1}{\varkappa_1-1}, \quad \vartheta_2 = \frac{1}{\varkappa_2-1}\right).$$

Here the values $\lambda_1\lambda_2$ and ε are determined as functions of the parameter n from the integrals of motion (7). The values of the quantities

$$Q_2(0) = f_2\varkappa_2(0)\left(\frac{2}{\varkappa_2(0)+1}\right)^{\chi}\frac{p_{10}}{\sigma}\frac{q[\lambda_2(0)]}{a_2^*(0)},$$

$$\left(\chi = \frac{\varkappa_2(0)}{\varkappa_2(0)-1}\right)\left(\begin{matrix}p_{10} = p_1(0)\\ T_{10} = T_1(0)\end{matrix}\right)$$

are given. The place at which the through-flow ceases $x = x_1$ and the corresponding value of $n = n_1$ are determined from the condition of equalization of static pressures in the pipes $p_1(x_1) = p_2(x_1)$, which is of the form

$$\varepsilon(n_1) = \sigma T^{1+\vartheta_1}(\lambda_1(n_1))\, T^{-(1+\vartheta_2)}(\lambda_2(n_1))$$

$$\left(\vartheta_2 = \frac{1}{\varkappa_2(n_1)-1}\right).$$

After the point $x = x_1$, the flow in the perforated pipes moves without through-flow with the constant parameters determined from formulas (7) at $n = n_1$.

The amount of gas injected into the second pipe is limited, moreover, by the speed of sound established in the second pipe, when $p_1 > p_2$. In order to achieve this flow regime with the given values of $\lambda_1(0)$, $\lambda_2(0)$, σ, τ, $\varkappa_1$, $\varkappa_2(0)$, $R_1R_2(0)$, β, η, it is essential that at the point $x = x_k$, where $\lambda_2(n_k) = 1$, the perforations between the pipes end. After the point $x = x_k$, the flow moves in isolated cylindrical pipes without perforations with the parameters determined by (7) on condition that $\lambda_2(n_k) = 1$.

The influence of the parameter β on the flow regime in perforated pipes is shown in Figs. 1 and 2 which present graphs of the variation of the parameters λ_1, λ_2 and ε as functions of n when $\beta = -1$, 0, +1 for gases with the same physical properties ($R_1 = R_2$, $c_{p1} = c_{p2}$, $\varkappa_1 = \varkappa_2 = 1.4$) and with the same initial stagnation temperatures ($\tau = 1$) at a given ratio of the initial total pressures and areas of the pipes ($\sigma\eta = 1$, $\sigma > 1$). In Figs. 1 and 2, curve 1 is the dimensionless flow velocity in the first pipe λ_1, curve 2 is the dimensionless flow velocity in the second pipe λ_2 with $\beta = +1$, curve 3 is the dimensionless velocity λ_2 with $\beta = 0$, curve 4 the dimensionless velocity λ_2 with $\beta = -1$, curve 5 the degree of compression ε in the second pipe when $\beta = +1$, curve 6 when $\beta = 0$, curve 7 the value of ε when $\beta = -1$. In Fig. 1, the quantities λ_1, λ_2, and ε are given for the case when the velocity is below the critical velocity in the initial sections of both pipes ($\lambda_1(0) = 0.906$, $\lambda_2(0) = 0.207$).

The flow in the first pipe is slowed with increasing n. When $\beta = 0$, +1, there is a drop in the total pressure along the entire length of the second pipe. When $\beta = -1$, there is an increase in the total pressure in the second pipe up to the time $\lambda_1 > \lambda_2$, and there is a decrease in the total pressure from the time $\lambda_1 < \lambda_2$. In this example, the mixing ends when the speed of sound is attained in the second pipe, after which the flow moves in pipes 1 and 2 separated by a solid wall. Figure 2 shows the graphs of λ_1, λ_2, and ε in the case when the velocity in the initial section of the first pipe is the speed of sound ($\lambda_1(0) = 1$) and that in the second pipe below the speed of sound ($\lambda_2(0) = 0.207$). In this case, there is an acceleration of the flow with growing n in the first pipe ($\lambda_1 \geq 1$). When $\beta = 0$, +1 there is a drop in the total pressure with increasing n; when the value $\beta = +1$, this drop is more pronounced than when the value $\beta = +1$ in Fig. 1. In this example, mixing with $\beta = 0$, +1 also ends with the speed of sound being attained in the second pipe. When $\beta = -1$, there is a monotonic increase in the total pressure with increasing n, for $\lambda_1 > \lambda_2$ is always true in this case. Mixing with $\beta = -1$ ends in this case with equalization of the static pressures in the pipes.

REFERENCES

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